Letters

Efficient Implicit Model-Predictive Control of a Three-Phase Inverter With an Output *LC* Filter

Muhammad Nauman and Ammar Hasan

***Abstract*—Three-phase inverters with an output *LC* filter are commonly used to provide sinusoidal voltages with low-harmonic distortion. Constraint on peak filter current is often desirable to protect the components from potential damage. In case of constraints, implicit and explicit model-predictive control (MPC) are some of the feasible controller options. The conventional implicit MPC requires a large number of computations, whereas explicit MPC cannot directly incorporate real-time changes in model parameters, while still being computationally expensive than some of the other control schemes, e.g., hysteresis, dead beat control, etc. In this paper, we propose a new approach to solve the optimization problem in implicit MPC that has a computational complexity approximately five times less than that of explicit MPC. We have been able to achieve lower computational requirements by exploiting the inverter model and the structure of constraints.**

***Index Terms*—Active set method, *LC* filter, model-predictive control, three-phase inverter.**

I. Introduction

THREE-PHASE inverters are commonly used to transfer energy from a dc source to an ac load. In applications such as uninterruptible power supplies and variable frequency drives, the three-phase inverters are commonly used with an output *LC* filter to provide sinusoidal voltages with low harmonic distortion [1]–[3].

Several controllers have been proposed in the literature to control three-phase inverters. Hysteresis and proportional integral (PI) controllers are commonly used for voltage regulation of three-phase inverters [4], [5]. Conventional stationary frame PI controller does not achieve zero steady-state error for a sinusoidal reference, and the error increases with an increase in reference signal frequency [6]. Synchronous dq reference frame PI controller [5] and proportional resonant regulator [6] attain zero steady-state error but cannot handle constraints. A drawback of hysteresis controller is variable switching frequency, which can reduce the efficiency of a converter due to an increase in switching losses [4]. Some other control schemes include deadbeat control and linear quadratic regulator (LQR) control. Deadbeat controller has been extensively used to control inverters [2], [4] and rectifiers [7], [8]. LQR provides a good dynamic performance [9]. However, none of the above-mentioned controllers can systematically handle constraint on peak filter current that is required to ensure protection of components of the inverter [10].

Model-predictive control (MPC) [1] has been extensively used in power electronics because of its flexibility to include constraints in the control scheme. However, a major drawback of MPC is that it requires a large number of online computations. To reduce the computational requirements, a variant called explicit MPC has been proposed [11]. In explicit MPC, the optimization problem is solved offline using multiparametric programming. Multiparametric programming yields a lookup table, which gives optimal control action as an explicit function of the state. This approach reduces the number of online computations. However, explicit MPC cannot directly account for real-time changes in the model, and its computational complexity increases exponentially with the number of states and constraints [12], [13]. In this paper, we propose an implicit MPC that solves the optimization problem online while having lower computational requirements than explicit MPC.

We have been able to achieve lower computational requirements than explicit MPC by using implicit MPC with a customized active set [14] approach for a single-step prediction horizon. In active set methods, a working set, which is a set of potential active constraints, is maintained and updated in each iteration of the algorithm. The computational complexity of active set methods grows exponentially with the total number of constraints. In the proposed scheme, we have been able to reduce the computational requirements by combining some of the constraints, thereby reducing the total number of constraints. We also use a single-step prediction horizon. Although it reduces number of computations for both the implicit and explicit MPC, in our case, it further aids us in reducing the number of constraints leading to an efficient optimization algorithm. While a single-step horizon is not common in MPC in general, the reason that it works for the three-phase inverter with an *LC* filter is that it is a stable minimum phase system that does not require a long prediction horizon. For this reason, the single-step prediction horizon can also be seen in other applications in power converters [15], [16]. Reducing the computational requirements is of interest since it allows the controller to operate at higher frequencies or alternately it allows for implementation on a cheaper hardware.

This paper is organized as follows. Section II deals with the converter modeling. Section III describes our problem formulation. The proposed scheme is discussed in Section IV. Simulations results for an inductive load are presented in Section VI. Section V compares the computational requirements of the proposed control scheme and explicit MPC.

II. Inverter Model

A three-phase inverter with an output *LC* filter for producing sinusoidal voltages is shown in Fig. 1. Load is assumed to be unknown and balanced [1]. Each leg of inverter has two switches that operate in complementary mode. The inverter has eight different switching states that can be represented by three binary

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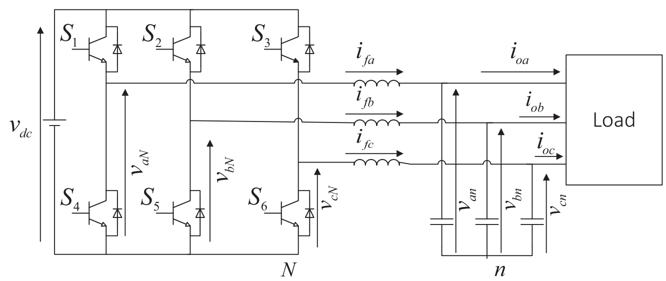

Fig. 1. Three-phase inverter with an *LC* filter.

switching signals, one for each leg of the inverter, stated below

$$S_a = \begin{cases} 1, & \text{if } S_1 \text{ is ON and } S_4 \text{ is OFF} \\ 0, & \text{if } S_1 \text{ is OFF and } S_4 \text{ is ON} \end{cases}$$

$$S_b = \begin{cases} 1, & \text{if } S_2 \text{ is ON and } S_5 \text{ is OFF} \\ 0, & \text{if } S_2 \text{ is OFF and } S_5 \text{ is ON} \end{cases}$$

$$S_c = \begin{cases} 1, & \text{if } S_3 \text{ is ON and } S_6 \text{ is OFF} \\ 0, & \text{if } S_3 \text{ is OFF and } S_6 \text{ is ON.} \end{cases}$$

Based on the above switching signals, we can find the following continuous-time state-space model of the inverter:

$$\frac{dx}{dt} = Ax + Bu_i + B_d i_o \tag{1}$$

with

$$i_o = \begin{bmatrix} i_{oa} \\ i_{ob} \\ i_{oc} \end{bmatrix}, \quad u_i = \begin{bmatrix} S_a v_{dc} - \left(\frac{S_a + S_b + S_c}{3}\right) v_{dc} \\ S_b v_{dc} - \left(\frac{S_a + S_b + S_c}{3}\right) v_{dc} \\ S_c v_{dc} - \left(\frac{S_a + S_b + S_c}{3}\right) v_{dc} \end{bmatrix},$$

$$x = \begin{bmatrix} i_{fa} \\ i_{fb} \\ i_{fc} \\ v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 0 & \frac{-1}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-1}{L} \\ \frac{1}{C} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C} & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{L} & 0 & 0 \\ 0 & \frac{1}{L} & 0 \\ 0 & 0 & \frac{1}{L} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{-1}{C} & 0 & 0 \\ 0 & \frac{-1}{C} & 0 \\ 0 & 0 & \frac{-1}{C} \end{bmatrix}.$$

where the currents and voltages are defined in Fig. 1. Output current is treated as a disturbance because of its dependence on an unknown load. Details of the modeling can be found in [1].

The model (1) is nonlinear because of the discrete nature of switching signals S_a, S_b, and S_c. The state-space averaging technique [17] is usually employed to approximate it with a linear model by utilizing the duty cycles of each leg of the inverter, instead of the switching signals. Using state-space averaging and discretization, we obtain the following discrete-time state-space model for sampling time T_s:

$$x_{k+1} = A_m x_k + B_m u_{i,k} + B_{dm} i_{o,k} \tag{2}$$

where

$$A_m = e^{AT_s}, B_m = \int_0^{T_s} e^{A\tau} B d\tau, B_{dm} = \int_0^{T_s} e^{A\tau} B_d d\tau,$$

$$u_{i,k} = \begin{bmatrix} v_{dc}(d_{a,k} - 0.5) \\ v_{dc}(d_{b,k} - 0.5) \\ v_{dc}(d_{c,k} - 0.5) \end{bmatrix}, \quad x_k = \begin{bmatrix} i_{fa,k} \\ i_{fb,k} \\ i_{fc,k} \\ v_{an,k} \\ v_{bn,k} \\ v_{cn,k} \end{bmatrix}, \quad i_{o,k} = \begin{bmatrix} i_{oa,k} \\ i_{ob,k} \\ i_{oc,k} \end{bmatrix}$$

where k is the sampling time index, and $d_{a,k}$, $d_{b,k}$, and $d_{c,k}$ are duty cycles of the PWM signal of respective inverter legs. In obtaining the above discrete-time model, we have also used the fact that at any instant, $d_{a,k} + d_{b,k} + d_{c,k} = 1.5$ for a balanced load [18].

III. Problem Formulation

A quadratic cost function to regulate the output voltages for a single-step prediction horizon is

$$J = \sum_{j=a,b,c} (v_{jn,k+1}^{ref} - v_{jn,k+1})^2 \tag{3}$$

where $v_{jn,k+1}^{\text{ref}}$ is the reference voltage. A single-step horizon is not usual in MPC for general applications. However, it is sufficient in our case since the three-phase inverter with an *LC* filter is a stable minimum phase system. For this reason, single-step or small prediction horizons are not uncommon in power converters [15], [16]. Using a single-step horizon helps us to simplify the optimization algorithm as discussed in the next section.

Using (2), cost function (3) can be represented in terms of the control variables $d_{a,k}$, $d_{b,k}$, and $d_{c,k}$ by the following equation:

$$J = \sum_{j=a,b,c} (v_{jn,k+1}^{ref} - \alpha_1 i_{fj,k} - \alpha_2 v_{jn,k} - \alpha_3 i_{oj,k} - \alpha_4 v_{dc}(d_{j,k} - 0.5))^2 \tag{4}$$

where α_1, α_2, α_3, and α_4 are constant coefficients depending on (2) and values of L and C. To avoid large currents, which can cause spikes in output voltages and damage the components, the inductor currents have to be bounded. The constraint on inductor currents can be stated as

$$I_{\min} \le i_{fj,k+1} \le I_{\max}, \quad \text{for } j = a, b, c. \tag{5}$$

To avoid impossible duty cycles and damage to components, the duty cycles are constrained as follows:

$$d_{\min} \le d_{j,k} \le d_{\max}, \quad \text{for } j = a, b, c. \tag{6}$$

As can be seen in (4)–(6), the cost and the constraints of each phase are independent of each other. Therefore, to simplify the controller design, we can focus on each phase separately. The optimization problem to find the control action for phase a is states as follows:

$$\min_{d_{a,k}} J_a = (v_{an,k+1}^{ref} - \alpha_1 i_{fa,k} - \alpha_2 v_{an,k} - \alpha_3 i_{oa,k} - \alpha_4 v_{dc}(d_{a,k} - 0.5))^2 \quad (7a)$$

$$\text{subject to} \quad I_{\min} \leq i_{fa,k+1} \leq I_{\max} \quad (7b)$$

$$d_{\min} \leq d_{a,k} \leq d_{\max}. \quad (7c)$$

Several algorithms exist in the literature to solve the optimization problem (7). Two main classes of optimization methods are interior point and active set methods. In the next section, we propose a computationally efficient algorithm that is tailored for (7) and based on the active set methods.

IV. Controller Design

The computational complexity of active set methods grows exponentially with the number of constraints. In the optimization problem (7), there are a total of four constraints, which are the upper and lower bounds on both the inductor current $i_{fa,k+1}$ and the duty cycle $d_{a,k}$. However, we can reduce the number of constraints by noting that the inductor current and duty cycle are related according to the discrete-time model (2). Using (2), we find the following relationship:

$$d_{a,k} = \frac{(i_{fa,k+1} - \beta_1 i_{fa,k} + \beta_2 v_{an,k} - \beta_3 i_{oa,k})}{\beta_4 v_{dc}} + 0.5 \quad (8)$$

where β_1, β_2, β_3, and β_4 are constant coefficients depending on (2) and the values of L and C. Since the relation of $d_{a,k}$ and $i_{fa,k+1}$ is monotonic, due to the linear expression and positive coefficient of $i_{fa,k+1}$, we can convert the constraint on inductor current (7b) to constraint on duty cycle as follows:

$$d_{I_{\min}} \leq d_{a,k} \leq d_{I_{\max}} \quad (9)$$

where $d_{I_{\min}}$ and $d_{I_{\max}}$ are the duty cycles computed from (8) for values of $i_{fa,k+1}$ equal to $I_{\min}$ and $I_{\max}$, respectively. Using (9) and (7c), all the constraints in problem (7) can be stated as

$$\max\{d_{I_{\min}}, d_{\min}\} \leq d_{a,k} \leq \min\{d_{I_{\max}}, d_{\max}\}. \quad (10)$$

In each iteration of active set methods, the Lagrange multipliers are computed and the working set is updated [14]. In our case, we only have two constraints, i.e., an upper and lower bound on the duty cycle (10). Moreover, these two constraints are also complementary, i.e., both the upper bound constraint and lower bound constraint cannot be active simultaneously. Therefore, to avoid calculations involved in updating the working set, we propose to compute the cost for all possible working sets and choose the duty cycle for the lowest cost that is feasible. There are three possible working sets: no constraint is active, the upper bound in (10) is active, or the lower bound in (10) is active.

When no constraint is active, the optimal duty cycle can be computed by using the derivative of the cost function (7a) with respect to $d_{a,k}$ and equating it to zero, which turns out to be

$$d_{a,k} = \frac{(v_{an,k+1}^{ref} - \alpha_1 i_{fa,k} - \alpha_2 v_{an,k} - \alpha_3 i_{oa,k})}{\alpha_4 v_{dc}} + 0.5. \quad (11)$$

Algorithm 1: Proposed Implicit MPC Algorithm

1: Measure or estimate $i_{fa,k}$, $v_{an,k}$ and $i_{oa,k}$
2: Compute $d_{I\min}$ by keeping $i_{fa,k+1} = I_{\min}$ in (8)
3: Compute $d_{I\max}$ by keeping $i_{fa,k+1} = I_{\max}$ in (8)
4: Compute $d_{a,k}$ using (11) for the unconstrained problem
5: **if** constraint (10) is violated
6: Compute J_1 (cost for $d_{a,k} = \max\{d_{I\min}, d_{\min}\}$)
7: Compute J_2 (cost for $d_{a,k} = \min\{d_{I\max}, d_{\max}\}$)
8: **if** $J_1 < J_2$
9: $d_{a,k} = \max\{d_{I\min}, d_{\min}\}$
10: **else**
11: $d_{a,k} = \min\{d_{I\max}, d_{\max}\}$
12: **end**
13: **end**

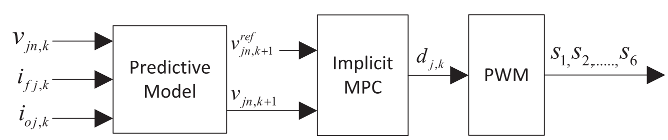

Fig. 2. Controller block diagram.

TABLE I
Proposed Controller Computations

Step No	Multiplications	Additions
Step 2	5	4
Step 3	5	4
Step 4	5	4
Step 6	6	5
Step 7	6	5
	Total Multiplications 27	Total Additions 22

However, we need to verify if the computed duty cycle is feasible, i.e., it should satisfy (10). For the other two working sets, we can find the cost by using $d_{a,k} = \max\{d_{I_{\min}}, d_{\min}\}$ or $d_{a,k} = \min\{d_{I_{\max}}, d_{\max}\}$. The proposed scheme for computing the optimal $d_{a,k}$ is given in Algorithm 1. The block diagram of the controller is shown in Fig. 2.

V. Computations

The number of computations involved in the proposed controller, i.e., Algorithm 1, are shown in Table I. The total computations required by the proposed algorithm for a single-step prediction horizon and a single phase are 27 multiplications and 22 additions.

To calculate the computations of the explicit MPC controller, it was implemented for the optimization problem (7) using the MPC tool box [19]. The multiparametric programming of explicit MPC generates 274 regions, leading to a binary search tree of depth equal to 9 [11]. Computations of explicit MPC controller for a single-step prediction horizon and single phase turn out to be 120 multiplications and 120 additions.

Comparing the computational requirements, it can be observed that the proposed algorithm is approximately five times faster than explicit MPC.

TABLE II
PARAMETERS FOR SIMULINK SIMULATIONS

Parameter	Numeric Value
Input dc Voltage v_{dc}	500 [V]
LC Filter Inductance L	1 [mH]
LC Filter Capacitance C	20 [μF]
Minimum Duty Cycle $d_{\min}$	0.1
Maximum Duty Cycle $d_{\max}$	0.9
Maximum Filter Current $I_{\max}$	12 [A]
Minimum Filter Current $I_{\min}$	-12 [A]
Sampling Time T_s	50 [μs]

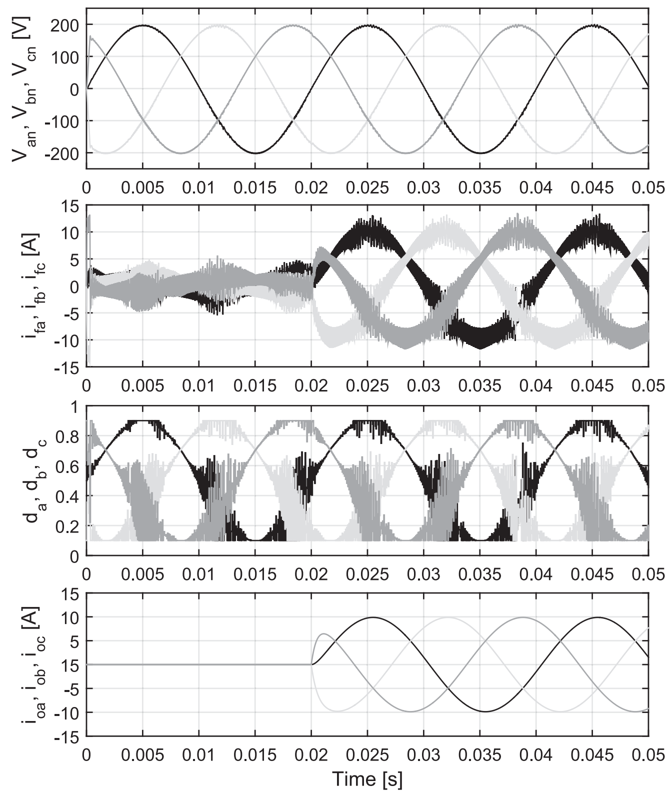

Fig. 3. Simulation results: Output voltages, filter currents, duty cycles, and load currents for inductive load.

VI. Simulation Results

The proposed scheme was simulated using Simulink with the parameters given in Table II. The output current was estimated using the observer in [1]. The performance of the proposed implicit controller for a series inductive–resistive load of 10 mH and 20 Ω is shown in Fig. 3. The load is applied at 0.02 s. It can be seen that the controller provides the desired output voltage while respecting the constraints on both the duty cycle and the inductor current. Initially, the filter capacitor tries to draw a large amount of current; however, it is constrained by the controller. It may be noted that since state-space averaging is used, which neglects the switching behavior, the filter current is expected to violate the constraints slightly [11].

VII. Conclusion

In this paper, a simple and tailored approach based on active set methods has been presented for an implicit MPC of a three-phase inverter with an output *LC* filter. The proposed scheme is computationally efficient as compared to the explicit MPC. The proposed scheme reduces the computational load by exploiting the structure of the inverter model and the constraints. Simulations have been performed to show that the proposed algorithm regulates the output voltages of the inverter subject to constraints on filter current and duty cycle. The reduced computational requirements could help operation of the controller at higher frequencies, implementation on a cheaper hardware, or a tradeoff between them.

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